

Acoustic and Thermoreactive Instabilities in a Photoionized Multiphase Medium

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Abstract. We study thermoreactive and acoustic instabilities in a diffuse gas, photoionized and heated by a radiation field. The analysis of the thermal instability by Field (1965) is extended to include the effects of the hydrogen recombination reaction, which, in general, is found to be a stabilizing agent for the condensation mode. This effect is stronger when the mean photon energy is not much larger than the hydrogen ionization energy. In addition, there are thermoreactive unstable equilibria for which an isobaric transition to a stable phase is not possible and the system evolves toward a dynamic state characterized by large amplitude, nonlinear periodic oscillations of temperature, density and hydrogen ionization fraction.

Acoustic waves are found to be unstable for some temperatures of both the cold and the warm phase of the ISM, also in regions where the gas is *thermally stable*, independently of the mean photon energy.

1. Introduction

Photoionization and radiative energy input processes are important in many different interstellar and intergalactic environments. Some examples are provided by Ly α clouds (Ikeuchi & Turner 1991, Charlton *et al.* 1993, Kulkarni & Fall 1993), outer disks of spiral galaxies (Corbelli & Salpeter 1993, Maloney 1993, Dove & Shull 1993), broad line emitting regions of AGNs (Kwan & Krolik 1981, Collin-Souffrin 1990), High Velocity Clouds (HVCs) in the Galactic halo (Songaila *et al.* 1989, Ferrara & Field 1994, McKee 1994) and the extended Galactic free electron layer (Reynolds 1993).

One of the interesting characteristics of photoionized objects is that, due to ongoing thermal instabilities, they often develop a multiphase structure. An example of a two-phase medium in which cold neutral medium coexists in pressure equilibrium with warm ionized medium is shown in Fig. 1, taken from Ferrara & Field (1994), which describes the ionization state of HVCs. A HVC in the Galactic halo is immersed in the extragalactic background radiation and the external layers of the cloud become ionized. For low values of the pressure of the external confining medium (for example, hot gas from a fountain) the cloud is optically thin, but when the pressure is increased, a cold central core develops in addition to the ionized interface. Fig. 1 displays the region of the cloud neutral hydrogen column density-size plane in which a multiphase structure exists.

Figure 1. Hydrogen column density of a HVC as a function of its size; the numbers refer to different values of the pressure in ergs cm^{-3} . *Solid* triangles refer to extragalactic ionizing field only, *open* triangles are for a composite extragalactic + free-free form hot halo gas. The upper dashed part shows the region in which HVC develop a multiphase structure.

Another important, though not fully understood, problem concerning the Galactic WNM (McKee & Ostriker 1977), is that the temperature of this phase, as tentatively derived from observations, is "too cool". The problem was firstly pointed out by Heiles (1989) studying the decomposition of the 21 cm emission line from prominent HI shells. He argued in favour of a component corresponding to a temperature ≤ 2000 K. More recently, different groups have contributed additional evidences along the same line, for other objects. Dickey *et al.* (1990) observed the outer disk of N1058, finding a constant velocity dispersion $\sigma \simeq 5.7 \text{ km s}^{-1}$ (equivalent to $T \simeq 4000$ K) well outside the Holmberg radius. Verschuur & Magnani (1994), have detected a wide component of the 21 cm line ($FWHM \simeq 12 \text{ km s}^{-1}$, *i.e.*, $T \simeq 3000$ K) towards high latitude Galactic HI.

An important question to address is the nature of this "anomalous" line broadening: is it thermal or turbulent? Both interpretations generate further problems. It is well known (Field 1965) that interstellar gas in this range of temperatures is thermally unstable. On the other hand, it is not clear what the source of turbulence in "quiet" regions, like outer disks and HVCs may be, and even worse, the lack of spatial variation in σ is hard to reconcile with such an explanation. In order to provide some possible solutions to the above problems we shall analyze in detail thermoreactive and acoustic modes in a photoionized ISM and shall find a number of new interesting physical effects which can be understood through a comparison with the classic Field (1965) paper. The consequences for the ISM will have to be carefully evaluated in a future work,

this contribution is based on a paper currently submitted for publication on ApJ (Corbelli & Ferrara 1994, thereafter Paper I).

2. Linear Instabilities: Acoustic and Thermoreactive Modes

We consider an ideal homogeneous gas with metallicity Z , ratio of specific heats γ , and in which hydrogen is the only reacting species. The basic equations are

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\rho \frac{d\mathbf{v}}{dt} + \nabla p = 0, \quad (2)$$

$$N_0 \frac{dx}{dt} - I(x, \rho, T) = 0, \quad (3)$$

$$\frac{N_0}{\gamma - 1} (1 + x) k_B \frac{dT}{dt} + N_0 \left(\frac{k_B T}{\gamma - 1} + \chi \right) \frac{dx}{dt} + L(x, \rho, T) - \frac{p}{\rho^2} \frac{d\rho}{dt} = 0, \quad (4)$$

$$p = N_0 \rho (1 + x) k_B T, \quad (5)$$

where, apart from usual symbols, x is the ionized fraction, N_0 is the Avogadro number and χ is the hydrogen ionization potential. The equations are similar to the ones used by Field (1965); however the reader may have noted the additional ionization equation (3). Ionization enters also the cooling function (coupling substantially the energy and ionization equations) and the pressure. Applying linear perturbations to the generic equilibrium state characterized by $T = T_0, \rho = \rho_0, v_0 = 0, p = p_0, x = x_0$, we obtain a *fourth* order dispersion relation of the form

$$\sum_{j=0}^4 a_j n^{4-j} = 0, \quad (6)$$

where the coefficients are given explicitly in Paper I. Instead of the three modes found by Field (one condensation + 2 acoustic wave modes), we have now four modes, almost always conjugate two by two, two of which correspond to thermoreactive modes, and two describing acoustic waves.

In addition, the inclusion of ionization modifies the isobaric criterion for thermal instability. A straight consequence of Field (1965) isobaric instability criterion, is that thermally unstable regions are characterized by a negative slope of the $p_0 - T_0$ curve. When L depends on x as well as on ρ and T , there is an additional term proportional to the temperature derivative of the ionization fraction

$$\left(\frac{\partial L}{\partial T} \right)_p = \left(\frac{\partial L}{\partial T} \right)_{\rho, x} - \frac{\rho}{T} \left(\frac{\partial L}{\partial \rho} \right)_{T, x} + \frac{\partial x}{\partial T} \left[\frac{\partial L}{\partial x} - \frac{\partial L}{\partial \rho} \frac{\rho}{1 + x} \right]_{\rho, T} < 0.$$

This criterion reduces to the Field's one when x is kept constant; in all other cases one cannot read the stability properties directly off the slope of the equilibrium curve.

Figure 2. Equilibrium curves p_0/ξ for four values of the metallicity Z . p_0 is in units of $\text{cm}^{-3} \text{ K}$ and ξ is the photoionization rate in s^{-1} . For each value of Z we show the equilibrium curves for three energies: in each panel the bottom curve is for $E = 15 \text{ eV}$, the middle curve is for $E = 40 \text{ eV}$ and the top curve is for $E = 100 \text{ eV}$.

To exemplify this important point we show in Fig. 2 different equilibrium curves in the usual plane $p/\xi - T$. The curves have been calculated for four different values of the metallicity in the range $Z = 0, 1$ and for three values of the mean photon energy of the radiation spectrum, $E = 15, 40, 100 \text{ eV}$. Higher values of E correspond to higher column densities, since low energy photons have a shorter mean free path for absorption. For every curve, regions which are linearly unstable (in the sense that at least one of the eigenvalues of the linearized system is positive, hence producing instability) are denoted by dotted lines. If, for example, one applies Field's criterion to the $Z = 1, E = 15 \text{ eV}$ curve, thermal stability would be inferred (since the slope is negative), contrary to the results of our analysis.

Fig. 3 summarizes the extension of stable and unstable regions in the $E - T$ plane (for several values of Z). A distinction is also made for acoustic and thermoreactive instabilities: dotted areas denote acoustically unstable regions, *i.e.*, only acoustic modes are unstable for small k . For low metallicity values ($Z \lesssim 0.1$) such regions do not exist; they first appear for $Z \simeq 0.5$ at low E . For $Z \geq 1$ two separate branches which are only acoustically unstable are found: one at low and one at high temperatures and they become wider as Z increases.

In the dashed regions the equilibrium is unstable for any k : except close to the low and high temperature boundaries next to the acoustically unstable regions, the most unstable modes are the thermoreactive ones for large k . Therefore we shall refer to these regions as thermally unstable regions. A pure hydrogen gas is thermally stable for any temperature and photon energy, but a small amount of metals induces the thermoreactive instability from roughly 100 K to 6000 K, even if they grow very slowly.

A major difference with respect to the standard thermal instability is represented by the presence of unstable acoustic waves in regions in which the gas is thermally stable. This occurrence is now allowed by the inclusion of the ionization. Besides, especially for intermediate values of the metallicity Z , thermal instability is now suppressed in some region of the parameter space E, T_0 .

3. The nature of the acoustic and thermoreactive instabilities

In order to better understand the nature of the two instabilities (acoustic and thermoreactive) found from the linear analysis, it is instructive to inspect the dispersion curves in a particularly exemplificative case. To this aim, we consider in the following the equilibrium state characterized by a density $N_0\rho = 1 \text{ cm}^{-3}$ (wavenumber and growth rates scale linearly with the density) and $E = 100 \text{ eV}, Z = 1$. From Fig. 3 it is seen for these values, temperatures $\sim 100 \text{ K}$ are only acoustically unstable for small k while if $T \sim 200 \text{ K}$ the equilibrium is unstable $\forall k$. Fig. 4 shows the dispersion curves relative to this case.

All three curves represent WW modes for small k (*i.e.*, $k \lesssim 10^{-18} \text{ cm}^{-1}$) and TR modes for large k ; TR (WW) modes in the isochoric (isobaric) regime are stable for all three cases. In the limit $k \rightarrow 0$, WW modes are oscillatory and unstable. For $T_0 = 120 \text{ K}$, n_r has a maximum at $\log k = -17.8$ followed by an abrupt break leading to stable TR modes. The minimum growth time for WW is $2.5 \times 10^7 (N_0\rho)^{-1} \text{ yr}$ (the curve shown has been multiplied by a factor of 10). Increasing the temperature TR modes become unstable for large k , and the curve for $T_0 = 160 \text{ K}$ represents a transition case between TR stable and unstable regimes. Except for this transition region, when TR modes are unstable, their growth rate is usually larger than the WW one (WW remain unstable anyway), and therefore they represent the most dangerous modes for the system. A more typical dispersion curve for unstable WW and TR modes is that shown for $T_0 = 200 \text{ K}$; as for the standard thermal instability n_r grows monotonically with k and the curve flattens out as $k \rightarrow \infty$. The effect of thermal conduction in this case would be to stabilize very large wavenumber perturbations ($k > k_c = 10^{-15} \text{ cm}^{-1}$, for the parameters considered).

The physical nature of the acoustic instability can be understood as follows. When a fluid element is compressed and rarefied by a travelling wave, there will be a net energy flow from the gas to the wave if γ increases from the compression to the expansion phase. If the ionization time is longer than the cooling time, compression will proceed at constant x and almost isothermally if the gas is stable, since this requires a steep dependence of the cooling function on T . Suppose that L depends on x in such a way that a lower x requires a higher T to provide the same cooling; on the ionization time scale, x will start to decrease

Figure 3. Unstable regions in the plane $E - T$ for various values of the metallicity Z . In the dotted regions there are unstable modes only for $k < k_m$ (acoustic waves). In the dashed regions equilibria have unstable modes $\forall k$ (they are both acoustically and thermoreactively unstable). The dashed curves at the bottom and left hand side of each panel limit the possible equilibrium values of T_0 for each value of the photon energy E .

Figure 4. Positive real part of growth rates n_r as function of the wavenumber k for three equilibrium temperatures relative to $E = 100$ eV, $Z = 1$, and $N_0\rho = 1$ cm⁻³. For $T_0 = 120$ K we plot the growth rate multiplied by a factor of 10. For this temperature the equilibrium is stable at large k while for $T_0 = 160$ and $T_0 = 200$ there is an unstable mode at all wavenumbers. The maximum growth rate for $T_0 = 200$ is for $k \rightarrow \infty$.

because of the higher density, temperature (and hence pressure) will increase destabilizing the wave. In other words, the ionization provides an effective increase of γ . As a consequence, the existence of the critical wavenumber for WW stabilization is fixed by the condition that the dynamical time is roughly equal to the ionization time. At that value of k the oscillation rate ($\propto k$ in the entire range) becomes much shorter than the ionization time and stiffening of the fluid caused by ionization is no longer possible.

For $T_0 = 120$ K Figure 4 shows that the system is acoustically unstable but thermoreactively stable; note, however, that without having considered the effects of ionization, it would undergo already to thermal instability at this temperature. For the equilibrium conditions we have considered *ionization is in fact a stabilizing agent* with respect to thermal instability when it introduces an oscillatory part in the growth rate. This oscillatory character often shown by thermoreactive instabilities can be illustrated with simple arguments. Suppose that the cooling is dominated by electron-metal impacts and to decrease slightly the temperature. Since TR modes are essentially isobaric, the density will then increase, and, if the initial state is thermally unstable, T will increase even further. However, on the ionization time scale (if not too large) the system will react decreasing x . If $L_x > 0$ (at constant pressure), the cooling will decrease, and as a consequence the temperature will raise back again, hence counteracting the instability and producing the unstable oscillatory motion (overstability). The previous condition $L_x > 0$ is always verified when cooling is dominated by electron-metal impact and $x \ll 1$, since in that case the heat-loss function

depends linearly on x . An increase of the metallicity Z in this unstable region rises the hydrogen fractional ionization and lowers the reactive times allowing the TR modes to become oscillatory.

The overstability of the TR modes produces interesting effects in the non-linear stages, which are the subject of the next Section.

4. Nonlinear evolution of TR modes

The aim of this Section is to investigate the nonlinear evolution of perturbations imposed to equilibrium states which are found to be thermoreactively unstable from the linear analysis. Since the results depend crucially on the phase structure of the gas, we classify the equilibria in two families: unstable multi-phase (*i.e.*, two or more isobaric equilibria of which at least one is stable) or unstable single-phase (*i.e.*, one or more isobaric unstable equilibria). When the photon energy is high ($E \gg 13.6$ eV) unstable equilibria are multi-phase; in the opposite case they are always single-phase (see Fig. 2). We will discuss the results using the equilibrium curve corresponding to $E = 40$ eV, $Z = 1$, which shows both equilibrium states depending on the value of p_0/ξ . The curve is reported in the top panel of Fig. 5 and has been expanded for convenience; solid (dashed) lines denote thermoreactively unstable (stable) equilibria. Each of the bottom panels shows the nonlinear evolution of perturbations (with amplitude $\sim 5\%$) imposed to the four equilibrium points P1-P4.

The first point $P1$ is a thermoreactively stable equilibrium with $T_0 = 120$, $x_0 = 0.012$ K. In the linear phase the perturbation shows oscillations with $n_i = 5 \times 10^{-6} \text{ yr}^{-1}$. These oscillations are clearly seen also during the nonlinear evolution before the perturbation is damped on a timescale $\sim 400\tau_h = 10^7 \text{ yr}$, where $\tau_h \equiv (5/2)kT/\xi E$ is the local heating time.

- *Multi-phase equilibria.* The unstable equilibrium $P4$ corresponds to $T_0 = 2700$, $x_0 = 0.18$ and is obtained when $\xi \simeq 3 \times 10^{-14} \text{ s}^{-1}$ for $N_0\rho = 1 \text{ cm}^{-3}$. The same value of p_0/ξ corresponds to the stable equilibrium point $T_0 = 8100$, $x_0 = 0.44$ with a value of the volume density about 3 times smaller. In the linear phase $P4$ is thermoreactively unstable with $n_i = 0$. The system reaches the other stable equilibrium after about $600\tau_h = 7.5 \times 10^6 \text{ yr}$ through (nonlinear) oscillations. This time is much longer than the typical time scale that one would have calculated without the ionization, and, indeed, the system spends a long time in a time-dependent, nonequilibrium state. This retarding effect, due to ionization and already discussed in the previous Section, occurs very often.

- *Single-phase equilibria.* Equilibria $P2$ and $P3$ in Fig. 5 correspond to single-phase equilibria. For $P2$, $T_0 = 150$ K and the perturbation is overstable in the linear phase with an oscillation rate larger than the growth rate. For this reason, a large number of oscillations of growing amplitude is seen in T, x, ρ before saturation takes place. After $t \sim 10^3\tau_h$ the system relaxes to a state of *periodic* oscillations of constant amplitude. Note, that the ionization lags the temperature, as expected. Far from the unstable/stable transition region n_i is zero or rather small compared to the growth rate. Therefore for $P3$ the perturbations grow to the saturated values in less than one oscillation period. After that moment, the evolution is qualitatively similar to that of $P2$ but the

Figure 5. *Top:* Expanded equilibrium curve $p_0/\xi - T_0$ for $E = 40$ eV and $Z = 1$; dashed-dotted lines refer to thermoreactively unstable equilibria; P1-P4 mark points for which nonlinear evolution is shown below. *Bottom:* Nonlinear evolution of the temperature and ionization fraction perturbations in the isobaric regime. Continuous curves refer to $\delta T/T_0$, dotted curve to $\delta x/x_0$ as function of time (normalized to τ_H). The density is set to $N_0\rho = 1 \text{ cm}^{-3}$.

temperature spans the very large range $1000 \lesssim T \lesssim 10^4$ K, since the oscillation is not centered around T_0 .

To conclude, isobaric TR instabilities produce much slower phase transitions than the usual thermal instabilities. When no other stable equilibrium is available the medium spends long time intervals in nonequilibrium states characterized by nonlinear periodic oscillations of temperature, density and ionization fraction.

5. Summary

We have studied acoustic and thermoreactive instabilities in a diffuse ISM heated and ionized by an external radiation field, of mean photon energy E and photoionization rate ξ , cooled by collisional excitation of metals and hydrogen lines. The addition of the hydrogen recombination reaction with respect to Field's classical treatment of thermal instability has several remarkable consequences especially when the ratio between the heat input and the ionization rate of the radiation field is rather small. In addition to the linear stability analysis we have investigated the nonlinear development of thermoreactive modes.

In the absence of metals the gas is stable. A very small amount of metals induces unstable acoustic and nonoscillatory thermoreactive modes and, as in Field's analysis, the fastest growing instability is the isobaric thermal mode. If the metal abundance is about half solar or higher the linear analysis shows that unstable modes often become oscillatory and ionization is a stabilizing agent, as explained in the previous Sections. Furthermore, a new type of situation arises in which the gas is thermoreactively stable but unstable acoustic waves develop on large scales. While for $Z \simeq 0.5$ this occurs only for optically thin media, with $E - \chi \ll 100$ eV, for larger Z two regions are found which are only acoustically unstable, one in the cold phase at $T \sim 100$ K and one in the warm phase at $T \sim 8000$ K.

Another new and important feature introduced by the hydrogen recombination reaction is that the stability of isobaric thermoreactive modes is no longer connected with the shape of the p_0/ξ curve. This implies that there are unstable equilibria which cannot make an isobaric phase transition to any other stable state. The nonlinear analysis has shown that in this case the fate of the gas is to evolve towards a nonequilibrium state characterized by periodic, nonlinear oscillations of density, temperature and hydrogen ionization fraction. This effect is especially important for small heat input/ionization rate ratios, and for $Z > 0.1$, but in general it would be interesting to extend this type of analysis to additional ionization and heating mechanisms (decaying neutrinos, dust grains).

A number of possible applications to astrophysical objects are presented in Paper I.

Acknowledgments. This work has benefited by a large number of inspiring and stimulating discussions with Prof. George B. Field. Daily contact with him has been a source of scientific and cultural enrichment, but above all, a unique human experience.

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